MEMSCON

Module on the Assessment of the Structural Condition of Reinforced Concrete Buildings

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Table of contents

List of Figures

List of Tables

1. Module on the Assessment of the Structural Condition

1.1 Introduction

The assessment of the structural condition of reinforced concrete buildings is based on an initial elaboration of records from a limited number of sensors that are transferred to the module. After the initial calculation in preprocessing phase of the Local Condition of each member, the module produces an input file for the commercially available software SAP2000-ETABS where the non linear structural analysis of the structural model of the building (Global condition) is performed. The results of the analysis are furthermore elaborated in a postprocessing phase in order to be obtained the actual values of the safety factors under operating loads or the damage indices under seismic forces for the totality of the structural members. The assessment concerns both the long term behaviour under operating conditions and the behaviour during a seismic event.

For the long term behaviour the usual reason for excess stress condition of the structural members are unforseeing differential settlements of the foundation of the columns. Their estimation is based on the records of 3 strain sensors per column instaled at their base cross section in the ground floor along longitudinal reinforcing bars at corner points. On the basis of the Sain Venant's assumption that the cross section remains plane after the deformation, the strains at three points are sufficient for the estimation of the strain at any other point of the cross section. The assessment of the local condition consists to the estimation of the axial forces and bending moments from the recorded strains for each one column. The assessment of the global condition consists to the evaluation of the differential settlements corresponding to the variation of the axial forces at the time interval between successive measurements. These values are transferred to the input file of the structural model as imposed displacements in combination with the dead and live loads. The safety factors for each one structural member result from the analysis.

The evaluation of the seismic behaviour is based on the installation of two 3D accelerometers at two extreme points of the slab of each storey and at the foundation level. The records of the accelerations are integrated twice with respect to time. The velocities of the points result from the first integration and the equivalent displacements from the second. These are transferred as imposed displacements to the input file of the structural model, in order to form a sequent of successive load cases that correspond to the time instances of the measurements, which are successively analysed. The results for each load case become the primary state for the next load case. The local condition consists to the estimation of the damage index of the end cross sections of the structural members based on energy dissipation criteria due to plastic rotations at those points for the bending moments resulted from the analysis. The condition for the global stability of each one storey is expressed by the ratio of the number of the developed plastic hinges at the end cross sections of the columns to the total number required for instability.

All the data required for the non linear analysis consisting of the geometry of the structure, the material properties, the type of the structural members, the dimensions of the cross sections, the reinforcement, the foundation conditions, the dead and the conventional live loads and the environmental actions are stored in the input file of the structural model. The additional data required for the elaboration of the records of the strain and acceleration sensors before their introduction to the input file of the structural model are defined in the respective following paragraphs.

The analytical relations and the procedures contained in the methodology report, which is the subject of the Deliverable 2.1, are incorporated in the pre- and postprocessing submodules and their interfaces with the non linear analysis program SAP2000-ETABS.

In the present report the final results concerning the structural assessment are presented in the form of tables available to the end user, as well as to constitute the data required for the Module on Rehabilitation options and Costs. An expert user can have approach to more detailed data and results from the input and output files of the non linear analysis of the structural model.

Because of the deficiency of the in field data, the testing of the module is performed on a virtual two storey building having plan view dimensions 8.00mX5.00m and 2x3.00=6.00m total height which is supported by six columns. From the results of the analysis of the virtual model, the module corresponding to the long term behaviour is tested. For the seismic behaviour, the part of the module concerning the estimation of the displacement from the records of the accelerometers is tested by using the data of the typical El Centro accelerogram, which describes a seismic ground motion. To test the rest of the module with regard to the seismic behaviour, records of the accelerometers installed on the floors are required which, however, are not yet available. For that reason the content of the corresponding tables remains temporarily blank.

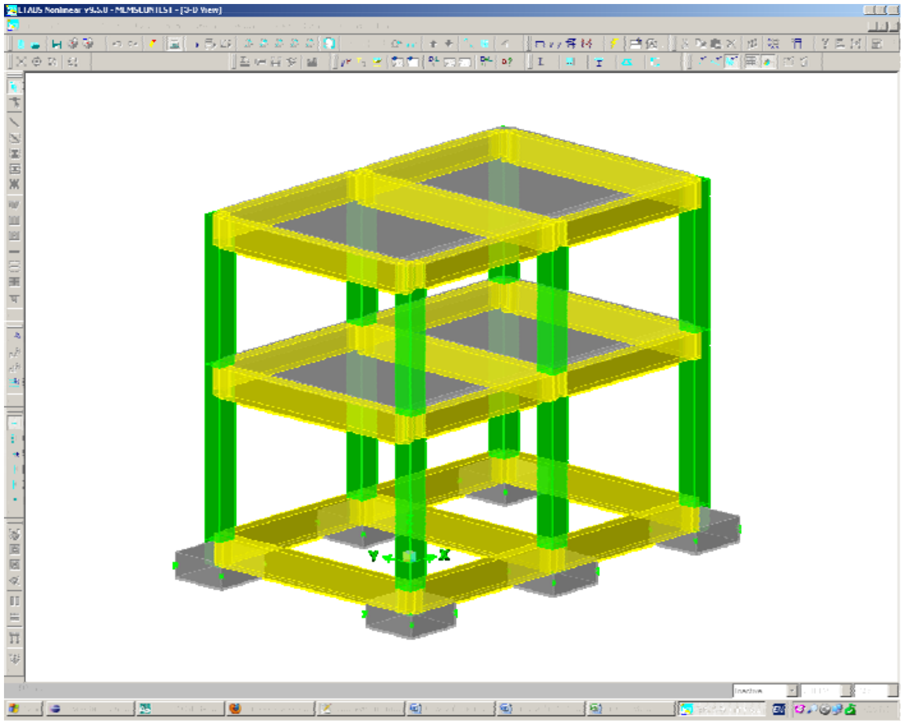

Fig. 1: 3D Space Frame Model of the Dummy Building

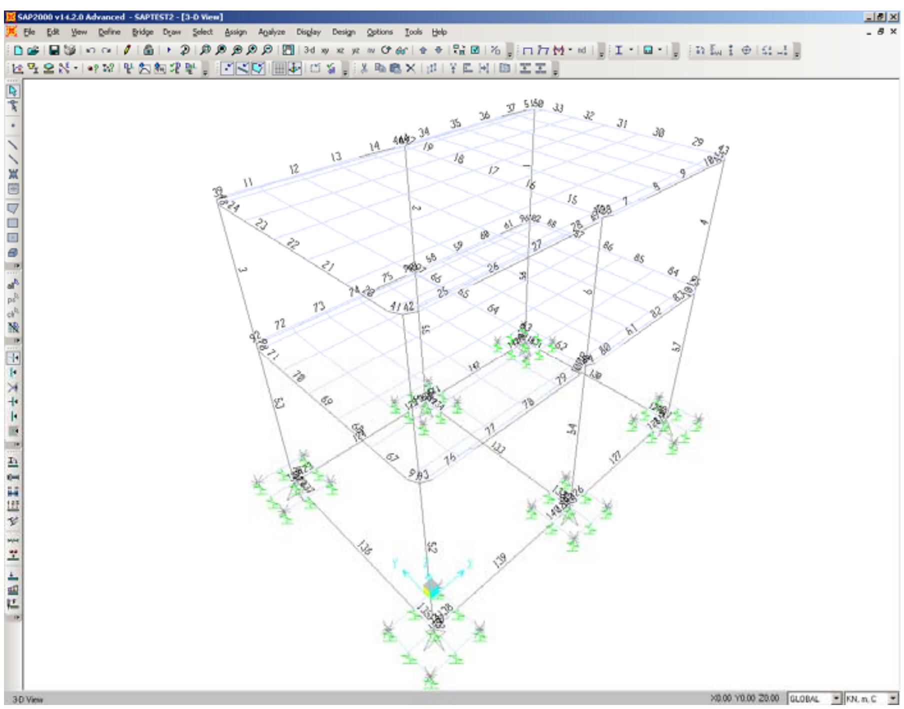

Fig. 2: 3D Model with Labels of the Structural Members

1.2. Long Term Behavior

1.2.1 Strain Sensors

The bottom cross section of every column at the ground floor is equipped with three strain sensors. The positions of these sensors in respect to the local coordinate system of the cross section are initially introduced by the end user and are shown on Table 1.

The "recorded" strains at the time of instance t_n are transferred from the database on Table 2. (Since there are no available infield data, the values of the table refer to virtual strains of the same model).

Table 1: Coordinates of sensors in the Local System of Columns Cross Sections (Initial Data)

	Sensor 1		Sensor 2		Sensor 3	
Column	x1	y1	x2	y2	x3	y3
52	0.16	0.26	-0.16	0.26	-0.16	-0.26
53	0.16	0.26	-0.16	0.26	-0.16	-0.26
54	0.16	0.26	-0.16	0.26	-0.16	-0.26
55	0.16	0.26	-0.16	0.26	-0.16	-0.26
56	0.16	0.26	-0.16	0.26	-0.16	-0.26
57	0.16	0.26	-0.16	0.26	-0.16	-0.26

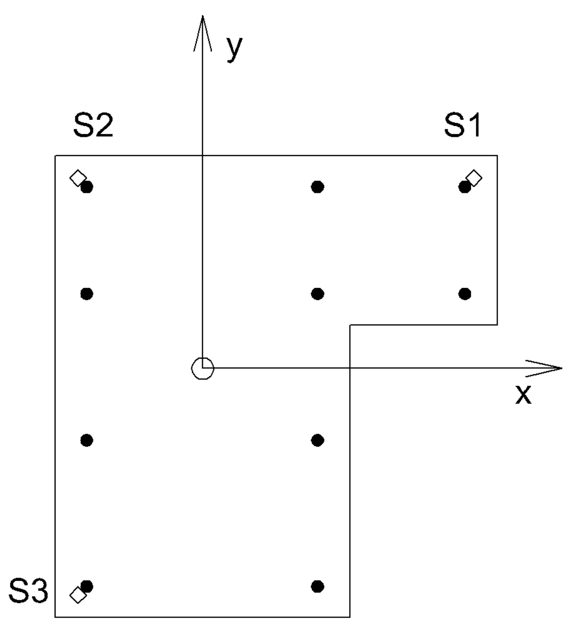

Fig. 3: Column Cross Section with Strain Sensors

Table 2: Recorded Strains εi(n) at time t(n) (Initial Data)
(n: measurement record number)

	Sensor 1		Sensor 2		Sensor 3	
Column	ε1(n-1)	ε1(n)	ε2(n-1)	ε2(n)	ε3(n-1)	ε3(n)
52	3.800E-04	5.500E-04	2.250E-04	4.730E-04	-3.090E-04	-4.080E-04
53	3.800E-04	5.500E-04	2.250E-04	4.730E-04	-3.090E-04	-4.080E-04
54	-5.020E-04	-5.500E-04	-3.710E-04	-3.850E-04	4.590E-04	4.400E-04
55	-5.020E-04	-5.500E-04	-3.710E-04	-3.850E-04	4.590E-04	4.400E-04
56	3.800E-04	5.500E-04	2.250E-04	4.730E-04	-3.090E-04	-4.080E-04
57	3.800E-04	5.500E-04	2.250E-04	4.730E-04	-3.090E-04	-4.080E-04

1.2.2 Local Condition

The recorded strains are inserted to the module and then the internal forces N, Mx, My of the columns at the bottom cross sections, where the strain sensors are installed are calculated according to paragraph 2.1.4 and 2.1.5 of the Deliverable 2.1. The ultimate internal forces N_R, M_{XR}, M_{YR}, corresponding to the state of maximum concrete strain $\varepsilon_c=\varepsilon_{cu}$ and are proportional to the actual forces are also calculated. The safety factor for each one cross section is defined by the relation

$$\gamma = \frac{N_R}{N} = \frac{M_{XR}}{M_X} = \frac{M_{YR}}{M_Y}$$

(eq. 1.2.2.1)

Table 3: Actual and Ultimate Internal Forces at the Base of Columns in Ground Floor

Column	Internal Forces - Actual			Internal Forces - Ultimate			Safety
C60/40	N	Mx	My	NR	MxR	MyR	Factor
52	-778.67	208.69	-13.60	-2658.15	712.41	-46.41	3.41
53	-778.67	208.69	-13.60	-2658.15	712.41	-46.41	3.41
54	-1280.71	-261.52	34.81	-3439.06	-702.26	93.47	2.69
55	-1280.71	-261.52	34.81	-3439.06	-702.26	93.47	2.69
56	-778.67	208.69	-13.60	-2658.15	712.41	-46.41	3.41
57	-778.67	208.69	-13.60	-2658.15	712.41	-46.41	3.41

1.2.3 Global Condition

The differential settlements developed at the base of the columns in the ground floor at time t_n are calculated from the variation of the axial forces at these points with respect to those developed at the time t_g of the completion of the structure according to the procedure described in paragraph 2.1.6 of the Deliverable 2.1. Furthermore they are separated to settlements due to the permanent loads G and those due to the conventional live loads Q, since different values of safety factors for permanent and live loads are provided by the codes.

Table 4: Differential Settlements of the Columns in the Ground Floor at the time tn.

Column	Ng	NQ,n	Nn	ΔNn	Δδ,n	ΔδG,n	ΔδQ,n
52	-584.00	-199.41	-778.67	-9.42	-0.0041	-0.0031	-0.0010
53	-619.15	-176.40	-804.09	11.45	-0.0032	-0.0024	-0.0007
54	-1011.76	-245.81	-1280.71	51.98	-0.0087	-0.0066	-0.0019
55	-840.87	-211.76	-1078.04	29.56	-0.0071	-0.0054	-0.0015
56	-568.43	-197.16	-778.67	-29.93	-0.0039	-0.0030	-0.0009
57	-748.20	-274.38	-1039.17	-53.64	-0.0036	-0.0027	-0.0008

where:

- Ng: Axial force due to dead loads (measurement at time tg)
- NQ,n: Axial force due to live loads at the time of the measurement n
- Nn: Axial force at time tn (measurement and calculation)
- ΔNn: $\Delta Nn = Nn - \zeta_n \times Ng$
- Δδ,n: Total Differential Settlement at time tn
- ΔδG,n: Differential Settlement at time tn due to dead loads
- ΔδQ,n: Differential Settlement at time tn due to live loads

The calculated differential settlements are transferred as imposed displacements to the input data of the structural model of the building. A non-linear structural analysis of the model, taking into account the existing reinforcement of the cross sections and the elastoplastic behavior of the materials, is executed by using the commercially available software SAP2000-ETABS for the following load combination, as specified by the codes:

$$\gamma_G(DL) + \gamma_Q(LL) + \gamma_G(\Delta\delta G) + \gamma_{Q\psi 1}(\Delta\delta Q) + \gamma_G(S) \qquad \text{(eq. 1.2.3.1)}$$

The internal forces at the end cross sections of the structural members results from the analysis and are presented on Table 5. The value of the PMMRatio corresponds to the ratio of the magnitude of the vector of the actual internal forces (axial force P and the bending moments M2, M3) to the magnitude of the corresponding ultimate internal forces and equals the inverse of the safety factor.

Table 5: Internal Forces

Frame	Station	OutputCase	CaseType	P	V2	V3	T	M2	M3	PMMRatio
Text	m	Text	Text	KN	KN	KN	KN-m	KN-m	KN-m	Unitless
52	0	COMB1	Combination	-189.70	-3.15	-5.17	0.00	-5.40	-2.90	0.056496
52	3.5	COMB1	Combination	-168.70	-3.15	-5.17	0.00	12.71	8.11	0.069227
53	0	COMB1	Combination	-189.70	-3.15	5.17	0.00	5.40	-2.90	0.056496
53	3.5	COMB1	Combination	-168.70	-3.15	5.17	0.00	-12.71	8.11	0.069227
54	0	COMB1	Combination	-298.15	0.26	-7.02	0.00	-6.27	0.11	0.088266
54	3.5	COMB1	Combination	-277.15	0.26	-7.02	0.00	18.32	-0.80	0.101714
55	0	COMB1	Combination	-298.15	0.26	7.02	0.00	6.27	0.11	0.088266
55	3.5	COMB1	Combination	-277.15	0.26	7.02	0.00	-18.32	-0.80	0.101714
56	0	COMB1	Combination	-184.88	2.89	4.80	0.00	5.25	2.91	0.055055
56	3.5	COMB1	Combination	-163.88	2.89	4.80	0.00	-11.55	-7.20	0.064

57	0	COMB1	Combination	-184.88	2.89	-4.80	0.00	-5.25	2.91	0.055055
57	3.5	COMB1	Combination	-163.88	2.89	-4.80	0.00	11.55	-7.20	0.064

1.3 Seismic Behavior

1.3.1 Seismic Displacements

Two accelerometers A_{m1} and A_{m2} are installed at two extreme points of each one storey m and at the ground slab over the foundation, having the same coordinates x_{a1}, y_{a1} and x_{a2}, y_{a2} in the horizontal directions of the global reference system for the totality of the storeys. During an earthquake motion the accelerometers generate the records of accelerations α_{m1x}, α_{m1y}, α_{m2x}, α_{m2y} for the sequence of the time instants tn. The horizontal displacements δ_{m1x}, δ_{m1y}, δ_{m2x}, δ_{m2y} result after applying a double numerical integration of the accelerations with respect to the time, according to the formulas:

Velocities: $$v_n = v_{n-1} + \frac{1}{2}(a_{n-1} + a_n)(t_n - t_{n-1})$$ (eq. 1.3.1.1)

Displacements: $$\delta_n = \delta_{n-1} + \frac{1}{2}(v_{n-1} + v_n)(t_n - t_{n-1})$$ (eq. 1.3.1.2)

Table 6: Accelerations and displacements at the positions of the two accelerometers in the storey m.

	Storey 1								
n	tn (x10^-3s)	a(11x)	δ(11x)	a(11y)	δ(11y)	a(12x)	δ(12x)	a(12y)	δ(12y)
1	0.00	0.279487	4.37E-07	0.084	1.31E-07	4.937	7.715E-06	-2.104	-3.288E-06
2	0.06	1.82466	4.60E-06	0.547	1.38E-06	-1.464	2.857E-05	6.157	-3.533E-06
3	0.12	1.975244	1.80E-05	0.593	5.40E-06	-4.283	4.588E-05	4.441	1.911E-05
4	0.18	1.428532	4.26E-05	0.429	1.28E-05	0.962	4.901E-05	4.739	7.266E-05
5	0.24	1.296392	7.68E-05	0.389	2.31E-05	-4.446	4.152E-05	5.735	1.569E-04
6	0.30	-0.030803	1.17E-04	-0.009	3.52E-05	-3.430	1.627E-05	-2.050	2.633E-04
7	0.36	-1.357998	1.58E-04	-0.407	4.73E-05	-4.407	-3.353E-05	-3.211	3.672E-04
8	0.42	-2.685193	1.89E-04	-0.806	5.68E-05	-0.612	-1.034E-04	-4.097	4.515E-04
9	0.48	-1.782085	2.08E-04	-0.535	6.23E-05	-2.564	-1.861E-04	0.973	5.195E-04
10	0.54	0.236127	2.17E-04	0.071	6.51E-05	-0.970	-2.793E-04	0.672	5.851E-04
11	0.60	2.254436	2.27E-04	0.676	6.82E-05	1.552	-3.771E-04	2.982	6.591E-04
12	0.66	1.623947	2.48E-04	0.487	7.44E-05	3.733	-4.657E-04	-4.635	7.361E-04
13	0.72	-0.330793	2.77E-04	-0.099	8.30E-05	-0.152	-5.405E-04	4.147	8.099E-04
14	0.78	-2.285632	3.03E-04	-0.686	9.09E-05	-0.552	-6.107E-04	-1.511	8.869E-04
15	0.84	2.854612	3.26E-04	0.856	9.79E-05	3.151	-6.780E-04	2.128	9.691E-04
16	0.90	0.877308	3.56E-04	0.263	1.07E-04	-4.926	-7.441E-04	-0.380	1.055E-03
17	0.96	0.179915	3.94E-04	0.054	1.18E-04	-2.964	-8.252E-04	-0.482	1.142E-03
18	1.02	2.042442	4.37E-04	0.613	1.31E-04	-0.901	-9.247E-04	5.790	1.236E-03
19	1.08	1.349758	4.88E-04	0.405	1.46E-04	6.158	-1.022E-03	-3.192	1.343E-03
20	1.14	0.359144	5.47E-04	0.108	1.55E-04	-2.715	-1.067E-03	-3.021	1.398E-03
	Storey 2								
n	tn (x10^-3s)	a(11x)	δ(11x)	a(11y)	δ(11y)	a(12x)	δ(12x)	a(12y)	δ(12y)
1	0.00	2.370292	3.70E-06	0.711	1.11E-06	3.812	5.956E-06	3.560	5.563E-06
2	0.06	1.714239	1.75E-05	0.514	5.25E-06	3.228	2.887E-05	-0.237	2.188E-05
3	0.12	1.55567	4.28E-05	0.467	1.28E-05	1.638	7.038E-05	5.064	5.093E-05

4	0.18	-0.036964	7.55E-05	-0.011	2.27E-05	3.382	1.273E-04	-1.683	9.281E-05
5	0.24	-1.629598	1.08E-04	-0.489	3.24E-05	-3.046	1.927E-04	-3.897	1.312E-04
6	0.30	-3.222232	1.30E-04	-0.967	3.91E-05	-4.133	2.473E-04	-0.293	1.544E-04
7	0.36	-2.138502	1.37E-04	-0.642	4.10E-05	-5.017	2.764E-04	-2.877	1.661E-04
8	0.42	0.283352	1.32E-04	0.085	3.96E-05	4.860	2.910E-04	4.537	1.754E-04
9	0.48	2.705323	1.29E-04	0.812	3.86E-05	-4.407	3.060E-04	-2.606	1.903E-04
10	0.54	1.948737	1.38E-04	0.585	4.13E-05	-4.033	3.086E-04	-4.012	1.980E-04
11	0.60	-0.396952	1.56E-04	-0.119	4.68E-05	-0.814	2.904E-04	-4.864	1.814E-04
12	0.66	-2.742758	1.72E-04	-0.823	5.17E-05	0.341	2.639E-04	2.364	1.470E-04
13	0.72	3.425534	1.84E-04	1.028	5.53E-05	-4.585	2.300E-04	-2.763	1.081E-04
14	0.78	1.05277	2.05E-04	0.316	6.14E-05	4.420	1.892E-04	-0.804	6.296E-05
15	0.84	0.215898	2.34E-04	0.065	7.02E-05	4.602	1.623E-04	-4.921	3.339E-06
16	0.90	2.45093	2.69E-04	0.735	8.08E-05	4.009	1.629E-04	-2.304	-7.652E-05
17	0.96	1.619709	3.15E-04	0.486	9.46E-05	0.690	1.843E-04	3.101	-1.664E-04
18	1.02	0.430973	3.71E-04	0.129	1.11E-04	1.003	2.157E-04	-1.651	-2.528E-04
19	1.08	-0.066041	4.30E-04	-0.020	1.29E-04	1.498	2.536E-04	-4.993	-3.473E-04
20	1.14	0.761413	4.91E-04	0.228	1.38E-04	1.102	2.746E-04	0.860	-3.998E-04

From the totality of the records are selected the time instants corresponding to pick values of differential displacements between successive storeys.

Table 7: Differential Displacements between successive storeys.

n	Δm1x	Δm1y	Δm1	Δm2x	Δm2y	Δm2	f(m,n)
1	1.134E-05	1.097E-05	1.578E-05	-1.759E-06	8.851E-06	9.024E-06	-2.520E-06
2	3.751E-05	4.883E-05	6.157E-05	2.948E-07	2.541E-05	2.542E-05	7.157E-06
3	4.911E-05	1.077E-04	1.184E-04	2.450E-05	3.182E-05	4.016E-05	4.732E-06
4	3.146E-05	1.870E-04	1.896E-04	7.832E-05	2.015E-05	8.087E-05	9.012E-06
5	3.628E-06	2.899E-04	2.899E-04	1.511E-04	-2.567E-05	1.533E-04	2.837E-05
6	-3.459E-05	3.837E-04	3.853E-04	2.310E-04	-1.089E-04	2.554E-04	5.108E-05
7	-9.209E-05	4.451E-04	4.545E-04	3.099E-04	-2.011E-04	3.695E-04	7.732E-05
8	-1.361E-04	4.907E-04	5.093E-04	3.944E-04	-2.761E-04	4.814E-04	1.020E-04
9	-1.475E-04	5.421E-04	5.619E-04	4.921E-04	-3.291E-04	5.920E-04	1.230E-04
10	-1.591E-04	6.132E-04	6.335E-04	5.879E-04	-3.872E-04	7.039E-04	1.437E-04
11	-1.882E-04	7.075E-04	7.321E-04	6.675E-04	-4.777E-04	8.208E-04	1.646E-04
12	-2.246E-04	8.254E-04	8.554E-04	7.296E-04	-5.892E-04	9.377E-04	1.835E-04
13	-2.675E-04	9.696E-04	1.006E-03	7.704E-04	-7.018E-04	1.042E-03	1.996E-04
14	-3.112E-04	1.138E-03	1.180E-03	7.999E-04	-8.240E-04	1.148E-03	2.137E-04
15	-3.444E-04	1.319E-03	1.363E-03	8.403E-04	-9.658E-04	1.280E-03	2.278E-04
16	-3.838E-04	1.495E-03	1.544E-03	9.070E-04	-1.131E-03	1.450E-03	2.482E-04
17	-4.310E-04	1.670E-03	1.725E-03	1.009E-03	-1.309E-03	1.653E-03	2.770E-04
18	-4.724E-04	1.862E-03	1.921E-03	1.140E-03	-1.489E-03	1.876E-03	3.102E-04
19	-5.198E-04	2.069E-03	2.133E-03	1.276E-03	-1.690E-03	2.118E-03	3.453E-04
20	-5.595E-04	2.173E-03	2.243E-03	1.341E-03	-1.798E-03	2.243E-03	3.655E-04

Choice criterion: $f_{m,n} = (\Delta_{m1x,n} - \Delta_{m1x,n-1})(\Delta_{m1x,n+1} - \Delta_{m1x,n})$ (eq. 1.3.1.3)

$f_{m,n}<0$ n selected

$f_{m,n}>0$ n rejected

The set of absolute displacements δm1x, δm1y, δm2x, δm2y corresponding to the selected time instants tn, are transferred to the input data of the structural model of the building as successive phases of the behavior by using the commercially available software SAP2000-ETABS.

1.3.2. Properties of Plastic Hinges

For the non linear analysis of the structural model of the building is considered the capability of the formation of plastic hinges at the end cross sections of the structural members, beams, columns and shear walls. The properties of the plastic hinges are introduced in the input file of the analysis with the values presented in the Table 8.

Table 8. Plastic Hinge Properties

$\frac{\varphi}{\varphi_y}$	0	0,25	0,50	0,75	1,00	1,50	2,00	2,50	3,00	3,50	4,00	5,00	6,00
$\frac{M}{M_y}$	0	0,68	0,90	0,98	1,00	0,96	0,87	0,78	0,69	0,61	0,54	0,42	0,33

The values of the maximum plastic bending moment M_Y and the corresponding plastic rotation φ_Y corresponding to the axial force N resulted from the last long-term measurement, are calculated by using a prepared external software according to relations of paragraph 2.2.4 of the Deliverable 2.1 and are transferred to the input file of the non linear analysis. It is

$$\varphi_Y = \frac{2}{3} h \kappa_Y \qquad \text{(eq. 1.3.2.1)}$$

Table 9. Plastic Bending Moments and Plastic Rotations

Member	End point	N	h_y	κ_{Yx}	φ_{Yx}	M_{Yx}	h_x	κ_{Yy}	φ_{Yy}	M_{Yy}
1	1									
	2									
2	1									
	2									
3	1									
	2									
4	1									

1.3.3. Local Condition

In the results of the non linear analysis for each one of the successive behavior phases corresponding to time instants t_n, the values of the plastic rotations $\varphi_{x't,n}$ and $\varphi_{y't,n}$ are contained at the plastic hinges in the local coordinate system x', y' of the structural members and is calculated the resultant plastic rotation

$$\varphi_{t,n} = \left(\varphi^2_{x't,n} + \varphi^2_{y't,n}\right)^{\frac{1}{2}} \qquad \text{(eq. 1.3.3.1)}$$

For the bending moments $M_{x't,n}$ and $M_{y't,n}$ and the resultant

$$M_{t,n} = \left(M^2_{x't,n} + M^2_{y't,n}\right) \qquad \text{(eq. 1.3.3.2)}$$

For the behavior phase t_n appearing the minimum value of the ratio $\frac{M_{t,n}}{\varphi_{t,n}}$, the maximum plastic bending moment M_Y and the corresponding plastic rotation φ_Y are calculated by a prepared external software.

In paragraph 2.25 of the Deliverable 2.1 for the definition of the damage index a modified version of the Parc – Ang damage criterion is used, expressed as the ratio of the dissipated energy due to the plastic deformation and the total available internal bound energy of the structural segment. The energy quantities are equal to the area of the hysteretic envelope, which is the diagram presenting the relation between the characteristic internal force in respect with its conjugate deformation. The following alternative considerations are presented, referred to different structural segments:

In the cross section level is considered the relation between the bending moment M and the plastic rotation φ. In the member level is considered the relation between the shear force V and the relative transversal displacement δ between the end cross sections of the member. Transformations of the above relations are furthermore presented, expressed in terms of the minimum stiffness of the structural segment resulting from the plastic deformation.

As more convenient for the present application, the consideration in the plastic hing level is selected, which is better adapted to the capabilities offered by the program SAP2000-ETABS. The corresponding expression for the hysteretic envelope which is compatible with a big number of experimental results is

$$M = M_Y\left(\frac{\varphi}{\varphi_Y}\right) exp2\left[1 - \left(\frac{\varphi}{\varphi_Y}\right)^{\frac{1}{2}}\right] \qquad \text{(eq. 1.3.3.3)}$$

The corresponding energy quantity is

$$E(\varphi)=\int_0^{\varphi} M d\varphi=\frac{3}{4}e^{2}M_Y\varphi_Y\left[1-\left[\frac{4}{3}\left(\frac{\varphi}{\varphi_Y}\right)^{\frac{3}{2}}+2\left(\frac{\varphi}{\varphi_Y}\right)+2\left(\frac{\varphi}{\varphi_Y}\right)^{\frac{1}{2}}+1\right]\cdot exp\left[-2\left(\frac{\varphi}{\varphi_Y}\right)^{\frac{1}{2}}\right]\right]$$

$$\cong e^{\frac{5}{2}}M_Y\varphi_Y\left[1-exp\left[-\frac{1}{5}\left(\frac{\varphi}{\varphi_Y}\right)^{\frac{4}{5}}\right]\right]$$

(eq. 1.3.3.4)

The damage index D will be:

$$D=\frac{E(\varphi)}{E(\varphi_u)}=\frac{1-exp\left[-\frac{1}{5}\left(\frac{\varphi}{\varphi_y}\right)^{\frac{4}{5}}\right]}{1-exp\left[-\frac{1}{5}\left(\frac{\varphi_u}{\varphi_y}\right)^{\frac{4}{5}}\right]}$$

(eq. 1.3.3.5)

Table 10. Damage Index *D*

Member	End point	φ	M_{xt}	M_{yt}	*M*	M_Y	κ_Y	φ_Y	D
1	1								
	2								
2	1								
	2								
3	1								
	2								
4									

1.3.4. Global Condition

Global instability of the structure results in the case where plastic hinges are developed at both and cross sections of the totality of the columns and shear walls of a single storey. Plastic hinges correspond to end cross sections with damage index $D \geq$ 0.20. If

n_m : The number of columns in the storey m

$2n_m$: The required number of hinges for instability

n_{hm} : The number of hinges develeved in storey m , the index of instability of the storey m is

$$D_{Gm}=\frac{n_{hm}}{2n_m}$$

(eq. 1.3.4.1)

And the index of the global instability of the building

$$D_G=\max D_{Gm}$$

(eq. 1.3.4.2)

Table 11. Index of Global Instability

Storey m	Number of Columns n_m	Number of Plastic Hinges n_{hm}	Instability Index D_{Gm}
1			
2			
3			
4			